MATH 162A Review: The Picard-Lindelöf Theorem

Facts to Know:

In differential geometry, we need to know certain quantity exists, and usually this would be equivalent to the existence to some ordinary differential equation. In differential equations, the Picard-Lindelöf theorem, also known as the Picard's existence theorem, Cauchy-Lipschitz theorem, or existence and uniqueness theorem gives a set of conditions under which an initial value problem has a unique solution.

For the equation $y' = f(x, y)$ with the initial value condition $y(t_0) = y_0$, if the function $f(x, y)$ is continuous, and the partial derivative function $f_y(x, y)$ is also continuous neat the point (x_0, y_0). Then there is an $\varepsilon > 0$, such that the solution exists on the interval $(t_0 - \varepsilon, t_0 + \varepsilon)$.